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NAIVE BLOWUPS AND CANONICAL BIRATIONALLY COMMUTATIVE FACTORS BY NEVINS AND SIERRA—REFEREE REPORT

In this interesting paper, Nevins and Sierra construct, for a large class of graded algebras A, a canonical homomorphism from A to a so-called Naive blowup B which is defined using the geometry of the point modules of the ring A. They also prove some interesting properties of this construction.

The idea for this goes back to Artin, Tate, and Van den Bergh's seminal paper, where a map like this is defined in great generality, but lands in a ring B in general which is hard to understand. In very nice cases, such as when A is strongly noetherian, the ring B is a twisted homogeneous coordinate ring of a scheme, which is a much easier ring to deal with, and the homomorphism is essentially surjective. In this paper the authors show that in a much wider set of circumstances one can prove that the homomorphism is essentially surjective onto a ring B which is a fairly well-behaved kind of subring of a twisted homogeneous coordinate ring (the naive blowup just mentioned). Compared to the earlier work on this subject, there are much greater technical obstacles which they need to overcome in order to prove this result. Such canonical maps have had many applications in noncommutative geometry in the past. Thus this is an important generalization since there are numerous examples of rings which are not strongly noetherian, and in any case it may be hard to decide if a ring is strongly noetherian or not.

In addition to the main theorem, the subsidiary result Theorem 4.10 is very interesting to me. It shows that any connected graded algebra, generated in degree 1, has a map to a birationally commutatative algebra B which is universal for maps to birationally commutative algebras. This is a nice purely ring-theoretic statement, which it seemed to me ought to have a more obvious ring-theoretic proof, but I couldn't find one. The authors prove this using their careful analysis of the geometry of the point modules of the ring.

The paper is very well-written and I have few comments for this reason. I only have three minor comments about the presentation, see below.

I think this paper is an important addition to the field and recommend it highly for publication in Math. Z.

In the definition of the point functor F on page 2, I found the definition in terms of every graded piece being a "rank 1 projective" a bit unclear. Later, on page 4, you also say this is the same as being flat with Hilbert series 1/(1-s). In either case I think it would help to make clear that one only considers modules which locally have the same rank at every point; otherwise the rank or the Hilbert series doesn't make sense.

In the statement of Theorem 1.2, part (1) of the conclusions, you reference the notion of birationally commutative algebra but this has not yet been defined. I think it would help to define this notion before this since it is hard to interpret this part of the theorem otherwise.

At the top of page 4, you define a module M to be bounded if $M_n = 0$ for all $n \geq n_0$. This is the usual definition of right bounded, not bounded.